Math Education: A Creative Approach

Veronika Irvine
Computer Science
University of Victoria
Victoria, BC Canada
vmi@uvic.ca

Melanie Tory
Computer Science
University of Victoria
Victoria, BC Canada
mtory@csc.uvic.ca

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Abstract

In our increasingly technology dependent society, Math education is becoming ever more important as a key to a successful career. To reach at risk students, we need to explore a variety of different approaches. While games have been highly successful with certain groups, the competitive nature is not attractive for all students. We propose that exploration of Math through Art be given increased attention. Major advancements in computer graphics and direct manipulation interfaces make it possible to create an Artist Studio micro-world in which math concepts ranging from Geometry and Algebra to Calculus can be explored in a creative manner.

Introduction

Math education is faced with a serious problem. As a society, we are becoming increasingly dependent on advanced technologies. A solid foundation in mathematics is critical to a career in science and technology and yet less than 80% of US high school students complete math courses beyond the mandatory level [5]. This predicament is even more dire as far as females are concerned. Up to the Grade 10 level, there is no significant difference in the Math Literacy test scores of females versus males [3] and yet women make up only 27% of Math and Computer Science Bachelor degree recipients in the US [4]. Lack of self-confidence and lack of engagement are considered to be the primary contributors to this gender imbalance.

There is a growing interest in using educational games to attract young students to math. While games can be powerful educational tools, the authors do not think they are the solution for everyone. For several decades, it has been recognized that a gender gap exists for video game users. A 2005 study looked at German adolescents (aged 12 to 19) and found that young women spend fewer hours playing video games (15% of females play at least once a week, 61% of males) and display less interest in video games (15% of females find video games interesting, 52% of males)[9]. The content of many video games — violence, aggressive actions and sexually provocative female characters — has often been cited as the reason for this gender difference, however, this is not a complete explanation. Changing the content is necessary but not sufficient to obtain gender equity in video games. A German study looked at the importance of competition in various game genres and the correlation to gender participation [9]. Highly competitive genres were found to be male dominated but in genres where competition is not a factor, males and females participated in equal numbers. Competitive genres were characterized by the need to win, time pressures, and the presence of threats or conflicts. This is consistent with research in the US that found women prefer puzzles and fitness games while men prefer action adventure games.

In contrast to gaming, making art is a creative, collaborative activity. The primary focus is self-expression, the communication of thoughts and feelings. In art, there are no mistakes; an art work is neither right nor wrong. The artists can explore a concept and decide, based on their own criteria, whether the results they are achieving meet their objectives. In addition to teaching the mechanics of a mathematical solution, we believe that the open ended, exploratory nature of artistic creation lends itself to teaching students to think like mathematicians. Through their own investigations and observations, students may come to see that math is not a closed book — they have the ability to pose and answer new and interesting questions.

The union of Math and Art is not a new idea. Euripides, in the 5th century BC, said "Mighty is geometry; joined with art, irresistible." My co-author and I propose to make use of the latest advances in technology — multi-touch interaction; SMART boards, tablets and smart phones; and non-photo-realistic rendering — to create a dynamic and expressive environment for artistic and mathematical exploration. Past work in this area has focused primarily on Geometry at the elementary school level. We argue that this educational approach can also be used for more abstract concepts such as Algebra and Calculus targeting high-school and middle-school students.

Related Work

The main evidence for the validity of our approach is Escher's World, a short, intense 12 hour workshop conducted at MIT in 1995[8]. Twelve high-school students explored symmetry and the operations of reflection and rotation using traditional art supplies (paper, paint, etc.) and computer programs including Geometer's SketchPad and professional graphics and image processing tools. The students worked individually and in groups and critiqued each other's work. There were several significant findings but we emphasize the following: 1) There was an increase (from 33% to 75%) in the use of imagery for solving mathematical word problems. 2) There was a 67% improvement in the positive perception of Math and 3) The findings were more pronounced for females over males.

There have been a number of computer programs designed for visualizing the intricacies of mathematics. One of the earliest and perhaps most widely used is LOGO and Turtle Graphics. Turtle Graphics uses a programming paradigm based on natural language to create geometric designs in Euclidean space. Geometer's Sketchpad, another successful example, uses concepts from the physical world, such as a straight edge and compass. Scripts are used to describe shapes according to their rules of behaviour. Both of these programs are particularly well suited for exploring symmetry and geometric transformations — design components with a great deal of aesthetic impact. While not designed specifically for creating art, they have been used for this purpose. Other tools with similar goals include Cinderella and Cabri.

Significant advancements in computer graphics have led to very sophisticated support for math visualization in tools like Mathematica and MatLab. The Wolfram Mathematica Blog frequently features artistic projects such as image manipulation using the Droste Effect. These tools are very advanced in the visualizations they can create but have a steep learning curve. Again, artistic creation is only loosely supported and requires a significant amount of supplemental material.

A few math education applications focus on crafts. DigiQuilt, used by elementary school students to explore fractions and symmetry, draws on the craft of quilting [6]. HyperGami allows the user to design and decorate polyhedra which can be printed and folded into 3 dimensional models [7].

Proposed Work

We propose a math education microworld that focuses on the creation and exhibition of art work. The content will be geared towards grade 8, 9 and 10 curriculum items related to computer graphics: Geometry, Trigonometry, Algebra and their precursors. The application should incorporate Smart Boards, Tablets, and Smart Phones. User interaction should be multi-modal using pens, paint brushes, physical objects and multi-touch gestures. The microworld will present various art projects in the form of artistic challenges based on mathematical constraints. As way of illustration, consider the following example adapted from a Wolfram Blog posting by Christopher Carlson (`www.blog.wolfram.com`). The artistic challenge shown in Figure 1 is to create a design using arcs of various sizes that begin and end on set of points.

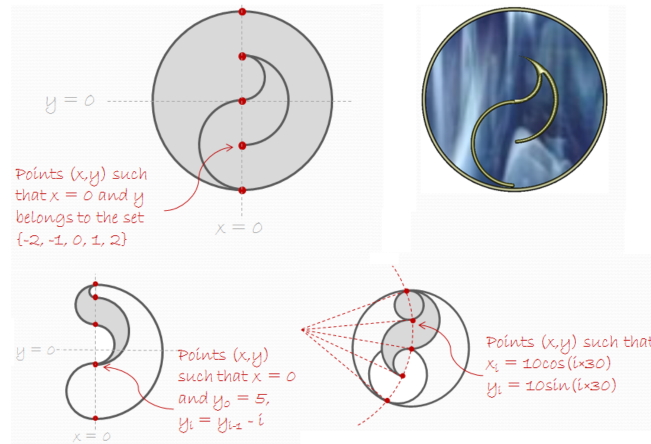

Figure 1: Top row shows sketch and finished art work for basic challenge. Bottom row shows sketches for two variations.

Artistic Challenge	
Grade K-8	Circle properties such as radius, arc, central angle Cartesian coordinates, integers
Grade 9	Translations, rotations, dilations; Simple polynomial expressions
Grade 10	Equation for a circle; equation with inequalities Solve problems using sine and cosine laws
Grade 12	Effect of translation and dilation on circle equation
Stained Glass Rendering	
Grade 8	Properties of visible light; how human vision works
Grade 9	Identify constants, coefficients and variables; rational exponents
Grade 12	Identify and manipulate vectors and scalars

Table 1: Math and Science curriculum items corresponding to example [1, 2].

The proposed microworld shall differ from currently available tools in several ways. 1) It will draw on practices from the world of art such as the portfolio and the ”critique”. 2) The notation for math expressions will be math symbols which will be edited using a pen-interface. Most existing tools use the keyboard and a programming language. 3) Some free-form input may be allowed. In most tools, the visualization is created as a script and then ”run” to see the result. In our design, some choices may be made through direct manipulation such as dipping a paint brush in a palette and stroking over a line. To gain maximum control over the design, the user will need to explore the math concepts behind the default choices. 4) Artistic rendering styles such as oil paint and stained glass will be available. Further, the physics and math behind these tools will be open and available for modification.

The proposed microworld shall consist of a framework and project modules. The framework will provide support for a variety of rendering tools drawn from state-of-the-art computer graphics techniques. User portfolios and skill level information will be archived. Skill level will be based on a progressive inventory of math concepts associated with projects completed and tools used. Determining whether a skill has been acquired may be based on a satisfaction survey completed by the student or teacher at the completion of a project.

A project module will provide the set of mathematical constraints that define the artistic challenge, as well as detailed descriptions and interactive examples. It is envisioned as something that can be shared between educators and should contain all the resources required to successfully recreate the design challenge.

The success of this microworld shall be evaluated by a user study in which at risk, female and indigenous students will be the target participants. This can be accomplished through outreach programs in classrooms and at after school clubs.

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